Abstract: Equilibrium real effective exchange rate estimate for the Slovak economy presented in this paper helps to evaluate the total impact of the development of nominal exchange rates against the currencies of Slovakia’s relevant trading partners, domestic and foreign inflation on the price competitiveness of domestic companies over time. The appreciating trajectory of the equilibrium exchange rate is related to the gradual nominal convergence of the Slovak economy to the developed countries that is founded on the macro-economic fundamentals development reflecting ongoing real convergence. On the basis of the equilibrium real exchange rate forecast resulting from the predicted values of the fundamentals, further real appreciation can be expected corresponding with the continuing convergence of the economy also in the future without jeopardising the economic balance. The real exchange rate misalignments contribute to the identification of expansive or restrictive monetary policy influence on the economy.

Keywords: real effective exchange rate, equilibrium exchange rate, Behavioural Equilibrium Exchange Rate, Fundamental Equilibrium Exchange Rate, cointegration, Slovak economy

JEL classification: F31, F37

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1 The views and results presented in this paper are those of the author and do not necessarily represent the official opinion of the National Bank of Slovakia.
Introduction

Equilibrium real exchange rate, as one of the components defining the real monetary conditions index RMCI, represents an important source of information about the monetary policy’s influence on the economy. Real exchange rate overvaluation has a restrictive influence on the economy through the deterioration of foreign trade balance, when the domestic producers’ price competitiveness is decreased and their production is replaced with foreign production on both the domestic and foreign market with a negative impact on the economic growth and employment. The opposite case of undervalued exchange rate leads to the overheating of economy and inflation growth. The pressure on necessary structural changes in the economy enabling more effective resources allocation is reduced in countries undergoing the transformation process. One of the possible uses of equilibrium exchange rate estimates is for example also the determination of fixed exchange rate in the case of a country entering a monetary union or when the monetary policy uses a credible foreign currency as nominal anchor. It is necessary to fix the exchange rate at a level that will not lead to the above described imbalances in the economy and result in distrust of foreign exchange market agents.

Equilibrium exchange rate is an unobservable variable, therefore its trajectory has to be estimated using approaches described in the literature (wide overview of individual approaches is offered by Williamson (1994), Driver & Westaway (2003) or Égert (2004)) differing in the definition of the analysed exchange rate, equilibrium state and time horizon for achieving the equilibrium, and also in the methods identifying the relations between macroeconomic variables and the exchange rate.

Purchasing Power Parity (PPP) represents one of the simplest approaches to the determination of an equilibrium exchange rate level. It assumes validity of the law of one price for homogenous goods in different countries expressed in a single currency, which implies the real exchange rate equal to 1 (absolute PPP) or a constant (relative PPP). The existence of heterogeneous goods and consumers preferences in individual countries, barriers against goods entering market in the form of transport and localisation costs, import taxes and quotas, but also the real convergence of countries undergoing economic transformation and Balassa-Samuelson effect, all lead to rejection of the one-price assumption in reality. Observed nonstationarity of a real exchange rate time series also violates the mentioned assumption.
**Statistical approaches** in the form of moving averages, trends or statistical filters of exchange rates time series extract its trend component and present it as equilibrium. Their main disadvantage is the inability to identify long run deviations of exchange rate from its equilibrium or to explain the equilibrium exchange rate path using macroeconomic variables. Therefore they do not allow an analysis of the impact of possible economic fundamentals changes on the equilibrium exchange rate. Their advantage of simplicity is facilitated mainly in preliminary estimates of the equilibrium exchange rate development.

**Single equation approaches** are based on a reduced form of various theoretical models, in which the exchange rate is a function of relevant economic fundamentals. The selection of fundamentals is influenced by empirical experience and recommendations in literature, data availability or specific character of the analysed economy. Depending on the observed data properties the parameters of a single equation model can be estimated with a range of methods ranging from OLS, through cointegration analysis, to the panel methods. The advantage of single equation approaches is relatively simple application and lower requirements on input database in comparison with multiple equation approaches, and also clearer interpretation of the resulting equilibrium exchange rate estimate. The absence of simultaneous relations between economic fundamentals is disadvantageous, because they might be helpful in explaining deeper causes of the specific equilibrium exchange rate trajectory. Another disadvantage is the risk of inconsistent exogenous variables development. The group of single equation approaches includes BEER (Behavioural Equilibrium Exchange Rate), in which the real exchange rate is a function of long-run economic factors selected without any former strict theoretical model background and medium-term factors represented by the uncovered interest parity (UIP) condition, PEER (Permanent Equilibrium Exchange Rate) decomposing the equilibrium exchange rate resulting from BEER approach into permanent and transitory components, monetary approach where the exchange rate depends on the transaction and speculative motivation for holding domestic and foreign currency or a reduced form of NATREX (Natural Real Exchange Rate) approach.

**Multiple equation approaches** range from simple models of foreign trade or current account of the analysed country influenced by the exchange rate up to complex global macroeconomic models enabling mutual influences of economies in the process of achieving their equilibrium states considering their global consistency (balanced international flows at the global level). The solution of these models searches for such exchange rate path that directs the modelled economies to internal and external equilibrium simultaneously. The complexity of larger scale models can become a disadvantage in their implementation and less clear inter-
pretation of equilibrium exchange rate estimation results. The multiple equation approaches include FEER (Fundamental Equilibrium Exchange Rate) defining as equilibrium such exchange rate that helps to achieve both the internal and external balance of the economy, DEER (Desired Equilibrium Exchange Rate) similar to FEER with the additional condition of optimal fiscal policy or NATREX (Natural Real Exchange Rate) that differs from FEER, which follows economic targets set for flow variables, by focusing on achieving long-run equilibrium of stock variables represented by foreign debt and accumulated capital.

There is no recommendation of a single best approach in the literature. In order to obtain an objective view on the equilibrium exchange rate, it is suitable to use various alternative approaches. Every equilibrium exchange rate estimate is influenced by the specific definition of equilibrium in the approach applied to obtain the estimate.

The objective of this paper is to apply the two most used approaches BEER and FEER to estimate equilibrium real effective exchange rate for the Slovak economy. Theoretical description of BEER and FEER approaches in part 1 is followed by their application on real effective exchange rate of Slovak economy to the 15 most important trading partners based on the PPI-manufacturing index in part 2. The last part 3 summarises and interprets the results of the equilibrium exchange rate estimation process.

1. Theoretical background of BEER and FEER approaches to equilibrium exchange rate estimation

BEER and FEER approaches represent most often applied methods for equilibrium exchange rates estimation. Because of this they were chosen to estimate the equilibrium development of effective exchange rate compiled for the Slovak economy. That selection leads to the need to describe those approaches in more detail in the following part of the paper.

**BEER (Behavioural Equilibrium Exchange Rate)** by authors Clark and MacDonald (MacDonald, 2000 or MacDonald, 2007) is a representative of the single equation approaches to the equilibrium exchange rate estimation. It is a reduced form model that can be described for period \( t \) with following equation, which distinguishes factors influencing the real exchange rate according to the time (Frait & Komárek, 1999):

\[
r_t = \alpha^T Z_{1t} + \beta^T Z_{2t} + \gamma^T T_{it} + u_t
\]
Equilibrium real effective exchange rate estimation for the Slovak economy

\( r \) – vector of the observed real exchange rate values
\( \alpha; \beta; \gamma \) – vectors of the model parameters
\( Z_1 \) – matrix of the long run fundamental determinants of the exchange rate
\( Z_2 \) – matrix of the medium run fundamental determinants of the exchange rate with the influence within the span of one business cycle
\( T \) – matrix of the variables with short run temporary influence on the exchange rate
\( u \) – vector of random disturbances

Short run equilibrium real exchange rate \( r^A \) is determined by the actual values of long-run and medium-run fundamentals:

\[
  r^A_t = \alpha^T Z^A_{1t} + \beta^T Z^A_{2t}
\]

The difference between the real exchange rate observed value and its equilibrium level is caused by the temporary impact of short run factors \( T \) and random factors \( u \) influencing the nominal exchange rate and price level in the domestic and foreign economy, for example short run speculative capital flows strongly impacted by volatile sentiment of foreign exchange market agents resulting from the recent political and economical events, central banks interventions, price liberalisation, changes in indirect taxes or seasonal factors.

Long run equilibrium real exchange rate \( r^L \) is based on the equilibrium values of fundamentals \( Z^L_1 \) and \( Z^L_2 \) sustainable in the long run.

\[
  r^L_t = \alpha^T Z^L_{1t} + \beta^T Z^L_{2t}
\]

The long run equilibrium values of the fundamentals are often estimated using Hodrick-Prescott filter to smooth the individual time series in practical applications.

The exchange rate is a function of fundamentals resulting from economic theory and corresponding parameters in the model. MacDonald includes the following economic variables among the fundamental determinants contained in the vector \( Z^L_{1t} \): Balassa-Samuelson effect and its various approximations (labour productivity differential between the tradable and nontradable sector, domestic and foreign total labour productivity ratio defined as GDP per employee or capita, CPI to PPI ratio), terms of trade, final consumption of government to GDP ratio, private and public savings and investment determining the net export, net foreign assets, foreign debt, trade balance, openness of the economy or FDI inflow.
The content of vector $Z_{2t}$ originates in the uncovered interest parity condition that defines the real interest rate differential as the sum of expected real exchange rate depreciation and risk premium.

\[
r^t = r^{e\,t+k} - (i_t - i^*_t) + \sigma_t,
\]

where

- $r^{e\,t+k}$ is the expected value of the real exchange rate in period $t+k$.
- $i_t - i^*_t$ is the real interest rate differential.
- $\sigma_t$ is the risk premium.

The expected value of the real exchange rate $r^{e\,t+k}$ is determined by the above-mentioned long run fundamentals from the vector $Z_{1t}$ in the model. Medium run factors consist of the domestic and foreign real interest rate.

Cointegration analysis (for example, Johansen cointegration test) is used to estimate parameters of the resulting single equation model. Cointegration enables to find a long run equilibrium relation between the real exchange rate and relevant macroeconomic fundamentals in a situation when the time series of observed variables are nonstationary, but they have a common stochastic trend.

**FEER (Fundamental Equilibrium Exchange Rate)** belongs to the multiple equation approaches to the equilibrium exchange rate estimation. Its author John Williamson defines the equilibrium exchange rate as a level of the real exchange rate that corresponds to internal and external economic balance simultaneously (Williamson, 1994). Internal balance is represented by a state when the production of domestic and foreign economy is at its potential level (corresponding to the non-accelerating inflation rate of unemployment - NAIRU). It means that the economy uses all its resources at such level that does not induce excessive inflation rate. External balance is defined as a normative target value for the current account of the balance of payments. Influences of the capital and financial account on the external balance can be taken into consideration when setting the target value for the current account balance. For example, sustainable surplus of the capital and financial account would allow to achieve higher deficit of the current account while still maintaining the external balance of the economy. For the sake of simplicity, only the trade balance can be taken into account. It is generated by net export of goods and services that is directly influenced by the real exchange rate development. The normative character of the FEER approach results from the above-mentioned fact and the equilibrium exchange rate level depends on the current account balance considered to be optimal (sustainable in the medium run) during the analysed period.
Two concepts exist under the FEER approach:

- **Use of a multi-country econometric model**
  Model of this type captures the international economic relations of all included countries, which enables, in the case of internal balance, to take into account the influence of exchange rates and external factors on the potential output development in individual countries. In the case of external balance, the current account target values for individual countries are set in order to keep their consistency at the global level (surplus in certain countries is balanced with a deficit in the rest of the countries, as a result, the global foreign trade is balanced). The equilibrium exchange rate path is obtained from such model through simulation under the condition of normatively set external balance state. Significant model complexity and complicated interpretation of the results represent the main disadvantage of this concept.

- **partial equilibrium approach (Rubaszek, 2003)**
  The estimation of equilibrium exchange rate level in this concept consists of three steps:

  1. Estimation of foreign trade equations, in which the functional dependence between export or import, exchange rate and foreign or domestic demand is assumed. These equations are connected through the net export identity and after possible adjustment for the balance of income and current transfers generate the current account equation.

  2. Determining the internal balance – estimation of domestic and foreign economy potential output that is exogenous to the model (it is abstracted from the feedback between exchange rate and potential output – a disadvantage of partial equilibrium approach). After substituting the potential output of domestic and foreign economy into the current account equation we obtain the “underlying current account equation” for the current account that corresponds to the situation when domestic and foreign economy are in the state of internal balance.

  3. Setting the external balance in the form of a specific current account level. There are three most often used methods for determining this equilibrium level:
     - the assumption of constant equilibrium current account to GDP ratio for a country,
     - the use of macroeconomic identity $CA = S - I$, where the current account balance is equal to the difference between total savings and invest-
tment (macroeconomic balance approach). Equilibrium level of the current account is dependent on the long run determinants of savings and investment (the degree of economic development, demographic factors or impacts of economic policy).

- the solvency criterion assessing the stock of accumulated foreign debt. It requires the stabilisation of accumulated foreign debt to GDP ratio at an equilibrium level sustainable in the long run. It is possible to take into account expected capital flows and GDP growth rate, too. Countries with fast GDP growth, relatively low foreign indebtedness or expected inflows of long term capital from abroad (FDI) are able to achieve higher sustainable current account deficits in comparison with countries with high level of foreign indebtedness or the risk of capital outflows. The condition for sustainability of the relatively high current account deficits in transforming economies is not to use the induced capital inflows for consumption but for capital accumulation (investment in technology modernisation), which enables to increase competitiveness and economic growth in order to secure the future servicing of the foreign debt and FDI revenues.

The equilibrium real exchange rate can be obtained from the “underlying current account” equation through fulfilling the external balance condition. The advantage of partial equilibrium approach is its relative simplicity and interpretability.

The partial equilibrium approach can be described by the following simple economic model:

\[
\begin{align*}
\text{foreign trade block} & \quad X = f(Y^*; RER) \\
\text{internal balance} & \quad Y = \bar{Y} \\
\text{external balance} & \quad CA = \bar{CA}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial X}{\partial Y^*} > 0; & \quad \frac{\partial X}{\partial RER} > 0 \\
\frac{\partial M}{\partial Y} > 0; & \quad \frac{\partial M}{\partial RER} < 0
\end{align*}
\]

RER – real exchange rate (index growth represents depreciation)
Equilibrium real effective exchange rate estimation for the Slovak economy

X – export
M – import
CA – current account identical to the trade balance in the simple model
\( \overline{CA} \) – current account equilibrium level
\( Y \) – domestic economy output
\( \overline{Y} \) – domestic economy potential output
\( Y' \) – foreign economy output
\( \overline{Y}' \) – foreign economy potential output

Graphical presentation in the space defined by domestic demand \( D \) and exchange rate \( RER \) (Graph 1) displays the determination of equilibrium exchange rate \( \overline{ERER} \) and corresponding level of domestic demand \( \overline{D} \) in the intersection of internal and external balance curves:

**Graph 1: Internal and external balance of the economy in the FEER model**

IB – internal balance; EB – external balance; \( \overline{ERER} \) – equilibrium real exchange rate

Internal and external balance curves divide the space defined by the output of domestic economy and the exchange rate into four quadrants:

I. positive output gap and current account deficit relative to a target
II: positive output gap and current account surplus relative to a target
III. negative output gap and current account surplus relative to a target
IV. negative output gap and current account deficit relative to a target
Changing potential output and equilibrium level of the current account shift the internal and external balance curves and determine the equilibrium trajectory of the exchange rate.

The interconnected targets for internal and external balance form a specific feature of the FEER model, because the net export is a component of the total output. Hence a foreign trade deficit manifests itself automatically in the GDP decrease, ceteris paribus. On the other hand, positive output gap caused by excessive domestic demand, for example, does not manifest itself only in an increased domestic price level growth leading to a decrease of real net export due to the appreciation of real exchange rate, but also in deterioration of foreign trade deficit through increased import in small open economies like Slovakia. A change in any tool variable in the FEER model (exchange rate or domestic demand) has therefore an impact on achieving both the external and internal balance of the economy also due to the mutual interaction of these two targets. It is necessary, of course, to use both tool variables to achieve both balances simultaneously.

2. Application of BEER and FEER approaches in equilibrium real exchange rate estimation for the Slovak economy

The object of the analysis is the real exchange rate against the 15 most important trading partners of Slovakia\(^2\) based on the PPI-manufacturing indicator compiled by the NBS. Growth of the REER index represents an appreciation as a result of its calculation facilitating the following formula (Gylánik, 2011):

\[
REER^t = REER^{t-1} \prod_{i=1}^{15} \left( \frac{E_i^t P^t}{P_i^{t-1}} \right)^{w_i}
\]

\(REER^t\) – real effective exchange rate in the present period
\(REER^{t-1}\) – real effective exchange rate in the previous period
\(E_i^t\) – bilateral nominal exchange rate of the domestic currency to the currency of country \(i\) in the present period expressed in the form of value quotation (the value of one domestic currency unit in the foreign currency)

\(^2\) Germany, Czech Republic, Italy, France, Austria, Poland, Hungary, United Kingdom, United States, The Netherlands, Belgium, Spain, Russian Federation, China and Republic of Korea.
The equilibrium real effective exchange rate estimation for the Slovak economy

\[ E_{i}^{t-1} \] – bilateral nominal exchange rate of domestic currency to the currency of country i in the previous period

\[ P^{t} \] – domestic price indicator (CPI, PPI, PPI-manufacturing, ULC) in the present period

\[ P^{t-1} \] – domestic price indicator in the previous period

\[ P_{i}^{t} \] – price indicator of country i in the present period

\[ P_{i}^{t-1} \] – price indicator of country i in the previous period

\[ w_{i}^{t} \] – weight of country i in the present period originating in the territorial structure of Slovak foreign trade turnover in sections 5-8 SITC (manufacturing goods)

The reason for the choice of PPI-manufacturing as the deflator in REER calculation is its independence from price changes related to administrative decisions (changes of indirect taxes or regulated prices) and to relatively higher volatility of agricultural products, commodities and energy prices that would complicate the parameters estimation of the econometric models in use and also the assessment of domestic producers’ competitiveness development representing one of the key purposes of equilibrium exchange rate estimates. The disadvantage of PPI-manufacturing indicator is lower comparability between countries and lower availability of both the historic and forecasted data relative to the broader defined price indices.

Monthly data on the REER based on PPI-manufacturing are converted to the quarterly frequency through the geometric average in order to correspond to the publication periodicity for the majority of macroeconomic data entering the equilibrium exchange rate models. The time series is then adjusted using the X12 method in order to estimate the parameters in models working with seasonally adjusted data.

**BEER approach**

The selection of economic fundamentals determining the real exchange rate path is influenced by empirical experience and recommendations in the literature referred to in the theoretical part of the paper, while considering the availability of relevant macroeconomic data for Slovakia and its trading partners. The assessment of individual variants of estimated equations is based on the conformity of parameter signs with the economic theory, statistical significance of the parameters and economic interpretability of the obtained results. This process has produced the following selection of fundamentals:

DPRODL – labour productivity differential between Slovakia and the weighted geometric average of its 15 relevant trading partners. Variable weights
are the same as those used in effective exchange rate calculation. Labour productivity is defined as the ratio of DP at constant prices to the number of employed persons in the ESA95 methodology. In the case of missing data for the number of employed persons, labour force sample survey data were used.

\[ \text{I}_Y \] – gross fixed capital formation to GDP ratio at constant prices

\[ \text{NX}_Y \] – net export to GDP ratio at constant prices

\[ \text{AFD}_X \] – ratio of foreign debt adjusted for window dressing and REPO trades to the nominal export in USD

According to economic theory it can be assumed that a rising labour productivity differential results in appreciation. Domestic economy is able to produce higher output with the same labour input relative to the foreign economy, therefore it is more competitive and able to absorb real exchange rate appreciation. An increase of investment to GDP ratio creates conditions for the introduction of modern more productive technologies enabling the production of more sophisticated goods that can be placed on the international market at stronger exchange rate or higher price. Higher net export to GDP ratio has a positive impact on the external balance of the economy that needs to be compensated with real exchange rate appreciation. Increased foreign debt to export ratio deteriorates the ability to cover the debt service with foreign currency income gained from export, the following real depreciation allows the improvement of the trade balance necessary to secure increased foreign debt servicing.

Quarterly seasonally adjusted time series covering the period of 1993 to 2010 obtained from the databases of OECD, IMF, ECB, World Bank and Eurostat were used to estimate the BEER model parameters. It was unavoidable to use linearly interpolated annual data in the case of missing quarterly data. The stationarity of individual fundamentals time series is tested at first. Based on the ADF test the variables real exchange rate, investment to GDP ratio and foreign debt to export ratio are integrated of order one I(1) at the level of significance \( \alpha = 1\% \). In the case of labour productivity differential and net export to GDP ratio, this test accepts trend stationarity or stationarity at the standard level of significance \( \alpha = 5\% \). However, the KPSS test rejects the stationarity of these time series at the same significance level. In the BEER approach there is a search for the long run equilibrium relation between the real exchange rate and economic fundamentals that is represented by a cointegration vector estimated using the Johansen test when the data are nonstationary. Two BEER models have been identified in the cointegration testing process (standard deviations of the parameters are presented in parentheses), with its output available in the appendix:
**Model 1**

\[
\ln \text{REER} = 0.54 \ln \text{DPRODL} + 0.13 \ln I_Y + 0.01 \ln \text{NX}_Y + 1.65 \\
(0.0386) \quad (0.0567) \quad (0.0014)
\]

Johansen test indicates the existence of two cointegration vectors at the level of significance \(\alpha = 5\%\), and one cointegration vector at the level of significance \(\alpha = 1\%\).

**Model 2**

\[
\ln \text{REER} = 0.80 \ln \text{DPRODL} + 0.01 \ln \text{NX}_Y - 0.16 \ln \text{AFD}_X + 1.72 \\
(0.1823) \quad (0.0040) \quad (0.0780)
\]

According to the Johansen test, single cointegration vector exists at the level of significance \(\alpha = 5\%\).

Estimated parameters in both models are significant and interpretable in line with the economic theory. The adjustment coefficients (presented in the estimation output in the appendix) are also significant and have correct signs, hence the real effective exchange rate converges in both ECM equations to a trajectory determined by the significant long run relation.

In order to estimate the equilibrium exchange rate development in the period of 2011 to 2013, it is necessary to know the predicted values of economic fundamentals. In this analysis such values are obtained from the medium term forecast of the NBS MTF-2011Q2 for the Slovak economy, from the predictions of EMU member countries sent to the ECB and from the forecast of economic development in countries outside the Economic and Monetary Union compiled by the ECB as part of the BMPE forecasting process finalised in June 2011, from the OECD Economic Outlook from December 2010 for trading partners of Slovakia not belonging to the euro area who are OECD members, and from the IMF World Economic Outlook from April 2011 for the rest of the partner countries. The future development of Slovakia’s foreign debt is estimated through an autoregressive process and trend\(^3\). The actual, forecasted and equilibrium values of individual fundamental determinants of the exchange rate are displayed in graph 2. The equilibrium labour productivity differential \(\text{DPRODL}_{eq}\) originates in the estimates of both domestic and foreign potential output and equilibrium number of employed persons consistent with NAIRU obtained from the above-mentioned sources. HP-filter with standard value of parameter \(\lambda = 1600\) for quarterly data is used in the case of

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\(^3\) \(\ln AF = 8.11 + 0.04 \ln \text{TREND}^3 + [\text{AR}(1)=0.94]\)
missing estimates for some countries and also to estimate the equilibrium path of the remaining fundamentals (designated with the suffix \_hp).

**Graph 2: Actual, forecasted and equilibrium values of the fundamentals**

![Graph 2](image)

Source: forecasts by the NBS, ECB and EMU member countries, OECD, IMF and own calculations

**Graph 3: Actual and short run equilibrium real effective exchange rate**

![Graph 3](image)

Source: own calculations

The result of the estimated BEER models is the short run equilibrium real effective exchange rate based on the observed and predicted values of macroeconomic fundamentals. Graph 3 captures its trajectory ERER\_a together with the actual development of the real effective exchange rate REER.

The fundamentals themselves are subject to random shocks and business cycle oscillations, therefore also the short run equilibrium exchange rate path is
volatile and represents only the short term equilibrium with complicated economic interpretation of its dynamics.

The long run equilibrium real exchange rate trajectory $\text{EREER}_l$ can be obtained after the substitution of equilibrium values of the real exchange rate macroeconomic determinants free of cyclical and random economic shocks influences into the estimated BEER models. It is depicted in graph 4 together with the actual REER development.

The economic interpretation of real effective exchange rate misalignments and the descriptive statistics of equilibrium exchange rate trajectories estimated using the BEER approach are presented in the following part together with the FEER approach results.

**FEER approach**

The application of FEER approach to estimate the equilibrium real effective exchange rate for the Slovak economy follows the partial equilibrium approach described in the previous theoretical part. It requires estimating foreign trade equations for Slovakia at first to quantify the dependences between the exchange rate, current account approximated with the net export for the sake of simplicity and the GDP. Based on these relations such exchange rate trajectory can be found that is consistent with the exogenously set internal and external balance of the economy.

Quarterly seasonally adjusted data obtained from the same sources as when constructing the BEER model are used to estimate the parameters in export and import equations. The time series entering the foreign trade equations represented by real export and import of Slovakia, domestic and foreign demand at constant prices, and real effective exchange rate are nonstationary ($I(1)$) at the level of significance $\alpha = 1\%$ according to the ADF test. The labour productivity differential is used as an additional explanatory variable in the export equation. Its not quite unambiguous stationarity tests results (stationarity according to the ADF test and its rejection by the KPSS test) are mentioned in the description of BEER model estimation. Individual equations are therefore estimated in the ECM form. In an
effort to estimate the long run equilibrium exchange rate trajectory, where the re-
action of exchange rate adjustments necessary to achieve balance in the economy
also through the short run relations causing higher exchange rate volatility is not
desirable, only the long run portions of estimated equations represented by the
cointegration vectors are used in the FEER model. The significance of long run
relations at the level $\alpha = 1\%$ is verified using the Johansen cointegration test, its
output is in the appendix. Estimated long run equations have the following form
(standard deviations of the parameters are presented in the parentheses):

**Export equation:**

\[
\ln X_{00} = 0.99 \ln M_{ef} - 1.31 \ln REER + 2.40 \ln DPRODL - 2.10
\]

\[
(0.2678) \hspace{1cm} (0.3354) \hspace{1cm} (0.7621)
\]

**Import equation:**

\[
\ln M_{00} = 0.48 \ln DD_{00} + 0.78 \ln REER + 0.49 \ln X_{00} - 3.56
\]

\[
(0.1963) \hspace{1cm} (0.2157) \hspace{1cm} (0.0698)
\]

$X_{00}$ – export at constant prices
$M_{00}$ – import at constant prices
$DD_{00}$ – domestic demand at constant prices calculated as the difference of
GDP and net export.
$M_{ef}$ – foreign demand defined as the weighted geometric average of real
imports of 15 most important trading partners of Slovakia. Variable
weights are based on the territorial structure of Slovak export in
sections 5-8 SITC.
$REER$ – real effective exchange rate to the currencies of 15 trading partners of
Slovakia based on the PPI-manufacturing indicator.
$DPRODL$ – labour productivity differential between Slovakia and the weighted
geometric average of its 15 relevant trading partners. Variable
weights are the same as those used in the effective exchange rate
calculation. Labour productivity is defined as the ratio of GDP at
constant prices to the number of employed persons in the ESA95
methodology. In the case of missing data on the number of
employed persons, the labour force sample survey data are used.
This variable allows to capture the dynamic growth of Slovak
export related to the FDI flows in the export-oriented companies
that cannot be explained with the usual variables (foreign demand
or exchange rate).
All the estimated long run parameters are significant and economically interpretable. Almost unit estimate of the export elasticity to foreign demand stabilises the share of Slovakia of the international trade. The improvement of foreign trade balance caused by foreign demand growth is partially offset through increased import of inputs by domestic exporters. If both domestic and foreign demand grows at the same rate, the foreign trade balance remains almost unchanged, *ceteris paribus*. The sensitivity of export on the exchange rate is higher in comparison with the import, because Slovakia is dependent on the import of those goods, in which its small open economy does not specialise, and also of commodities not present in its territory. Relatively high value of the parameter corresponding to the labour productivity differential reflects the strong impact of economic transformation and FDI inflow resulting in labour productivity growth on the export capability of Slovak economy.

The next step in FEER approach application is the setting of internal balance for the domestic and foreign economy. Internal balance is a state when the economy produces output at the level of its potential without any excessive inflation pressure or too high unemployment rate exceeding the NAIRU. The potential output estimate for the domestic economy was adopted from the NBS medium term forecast MTF-2011Q2. The equilibrium labour productivity differential $DPRODL_{eq}$ (depicted in graph 2) corresponding to it reflects also the equilibrium number of employed persons consistent with NAIRU and the weighted average of foreign labour productivities calculated from the analogical equilibrium variables for the 15 relevant trading partners obtained from the sources mentioned in the description of BEER model application. Equilibrium foreign demand $M_{ef\_hp}$ is estimated with the HP-filter with the standard value of parameter $\lambda = 1600$. Actual, predicted and equilibrium values of these variables are captured in graph 5.

**Graph 5: Internal balance of the domestic and foreign economy**

Source: Predictions by the NBS, ECB and EMU member countries, OECD, IMF and own calculations
In order to estimate the equilibrium exchange rate with the FEER approach, it is necessary to set the external balance represented by a normative sustainable target for the balance of payments current account. During the period of 1993 to 2006, the equilibrium level of the current account excluding the income balance is defined as the constant -3.5 % share of the GDP. This sustainable deficit originates in the results published in Doisy & Herve (2002) and Gavura, Ševčovic & Vodička (1999) that are based on deriving the sustainable current account balance from the solvency criterion for a country requiring foreign debt to GDP ratio stabilisation under certain assumptions about economic growth, exchange rate appreciation and capital inflows. The positive impact of FDI inflows (foremost of those in the form of new “green field” companies) on export started to manifest in 2007 and 2008 and the sustainable current account deficit to GDP ratio decreased to -2 %. Had the sustainable deficit not decrease, the external balance of the economy would have to be achieved at the cost of lower production, for example, by the remaining companies without any foreign investors intervention. The sustainable current account balance to GDP ratio for the period from 2009 until the end of forecast horizon is the outcome of recent analysis by Stračhotová (2011), in which capital flows are conditional upon the assumed development of foreign assets held by domestic residents, domestic liabilities held by foreign investors, real interest rates and economic growth lead to the identification of current account surplus (excluding the income balance) to GDP ratio at the level of 0.66 % stabilising the net international investment position. The following identity assigns corresponding real net export needed in the FEER model to the estimated sustainable current account excluding the income balance through accounting for the observed development of the remaining components of the balance of payments current account and for the influence of export and import deflators.

\[ \text{NX00\_eq} = \text{Y\_eq} \cdot \text{P\_y} \cdot \text{ca\_eq} - (\text{CA} - \text{BI} - \text{NX00}) \]

\text{NX00\_eq} – equilibrium real net export
\text{Y\_eq} – potential output of the economy
\text{P\_y} – GDP deflator
\text{ca\_eq} – sustainable current account excluding the income balance to GDP ratio
\text{CA} – current account of the balance of payments
\text{BI} – balance of income
\text{NX00} – net export at constant prices
The path of sustainable net export at constant prices to potential output ratio EB defining the external balance of Slovak economy is captured on the Graph 6 together with its trajectory EB_hp smoothed with HP-filter with the parameter $\lambda = 1600$. The filtering is unavoidable due to disequilibrium disturbances in the remaining components of the current account that influence the real net export needed to achieve sustainable development of the current account.

The FEER model is complete after the substitution of variables reflecting the internal balance of domestic and foreign economy into the export equation and after the addition of identities enforcing simultaneous achievement of both the internal and external balance of the economy into the foreign trade block. Export and import equations contain also the residuals obtained from their individual simulation that approximate factors not included among the explanatory variables. Since these factors are subject to economic disturbances too, the residuals were smoothed with the HP-filter with parameter $\lambda = 1600$. This prevents the exchange rate path shifting the economy to the equilibrium to be biased due to possible longer term differences between the observed and model generated foreign trade balance caused by abstraction from the short run parts of the ECM equations for export and import in order to obtain a less volatile long run equilibrium exchange rate trajectory without any influence of the short run relations. In the appendix there are charts comparing the observed, predicted (NBS forecast MTF-2011Q2) and FEER model generated values of the endogenous variables without residuals and after their inclusion into the model.

\[
\begin{align*}
\ln X_{00} &= 0.99 \ln M_{ef \_hp} - 1.31 \ln REER + 2.40 \ln DPRODL_eq - 2.10 + e_{X00 \_hp} \\
\ln M_{00} &= 0.48 \ln DD00 + 0.78 \ln REER + 0.49 \ln X_{00} - 3.56 + e_{M00 \_hp} \\
NX_{00} &= X_{00} - M_{00} \\
Y_{00} &= DD00 + NX_{00} \\
CA_{\_EB} &= EB_{\_hp} \cdot Y_{00} \\
y_{\_target} &= Y_{00} - Y_{\_eq} \\
ca_{\_target} &= NX_{00} - CA_{\_EB}
\end{align*}
\]
NX00  – net export at constant prices  
Y00  – GDP at constant prices  
CA_EB  – sustainable current account balance representing the economy’s external balance  
EB_hp  – share of the GDP determining the sustainable current account balance  
Y_eq  – potential output representing the economy’s internal balance  
y_target  – deviation from the economy’s internal balance, output gap  
ca_target  – deviation from the external balance expressed as the difference between sustainable current account levels and net export in the model simplification  
e_X00_hp  – smoothed influence of the factors not included among the variables explaining export  
e_M00_hp  – smoothed influence of the factors not included among the variables explaining import

The final model is solved for two target variables represented by the exchange rate and domestic demand that are used through their influence on the GDP and net export to achieve simultaneously two targets represented by the internal and external balance of the domestic economy corresponding to zero values of the deviation variables y_target and ca_target. It is suitable to mention in this context, that the selection of adequate FEER model for equilibrium exchange rate estimation has to consider not only the economic interpretability and statistical significance of the parameters or the ability of the model to fit historical data, but also the requirement for sufficiently large elasticities of export and import to the exchange rate. In the opposite case, the exchange rate as a relatively weak tool variable would have to change substantially to shift the economy to the equilibrium state. These pronounced changes would complicate the economic interpretation of equilibrium exchange rate development.

Graph 7 depicts the actual and forecasted path of the endogenous variables (export, import, net export and GDP) according to the NBS medium term forecast MTF-2011Q2 and of the tool variables (real effective exchange rate and domestic demand) designated with the suffix Base in comparison to their equilibrium development designated with the suffix feer_ef_11q2, in which the economy simultaneously achieves zero output gap and sustainable values of the net export.
The mechanism of shifting the economy to the equilibrium through the influence of changing trajectories of the two tool variables in the FEER model can be described by the example of financial crisis impact on the real economy of Slovakia since 2009. A negative output gap opened suddenly in the Slovak economy. Taking into account the external balance target representing lower surplus in comparison to the actual development, a higher domestic demand representing one of the tool variables is needed to close the output gap and achieve internal balance. But the increased domestic demand leads also to the import growth deteriorating net export together with the negative impact of halted labour productivity differential growth and slower increase in foreign demand on export. Therefore, the equilibrium real effective exchange rate representing the other tool variable achieves initially weaker levels in comparison to the actual development that are necessary to maintain the external balance of the domestic economy. After the gradual growth recovery of labour productivity differential and foreign demand corresponding to the internal balance of domestic and foreign economy the external balance can be achieved at stronger exchange rate levels relative to its observed values. Hence, the equilibrium state of the economy exists at the appreciated real exchange rate that leads to lower export and higher import in the forecasted period (except 2013, when the influence of lower export on the import related to it prevails) in line with the lower targeted foreign trade balance and at corresponding higher domestic demand needed to achieve the internal balance target represented by the potential output.
Further economic interpretation of real exchange rate misalignments is presented in the following part together with the results of the BEER model.
The equilibrium exchange rate trajectory is influenced by the normative choice of the external balance target for the economy in the FEER approach. In order to explore the sensitivity of the estimated FEER model to a change in this target, the equilibrium exchange rate estimate is repeated under the assumption of sustainable current account (excluding the income balance) to GDP ratio at the constant level of -3.5% during the whole analysed period. Graph 8 compares the original (designated with the suffix `feer_ef_11q2`) and alternative (designated with the suffix `eb3_5`) path of the sustainable net export and tool variables represented by the real effective exchange rate and domestic demand serving to achieve the internal and external balance of the economy.

**Graph 8: Alternative definitions of the external balance of the economy and corresponding trajectories of the real exchange rate and domestic demand**

The higher targeted deficit enables to achieve the external balance of the economy at appreciated real exchange rate. Its shift to the stronger levels is moderated by
The equilibrium exchange rate estimate identifies real effective exchange rate undervaluation at the beginning of the analysed period in 1993 and 1994 that is characteristic for several countries entering the transformation process with an initial currency devaluation. The real exchange rate moved close to its equilibrium in 1995 and 1996 after the initial undervaluation had faded out. Keeping the Slovak currency in the fixed exchange rate regime led to its growing real overvaluation during the next two years. Imbalances in the economy manifesting through the double deficit of foreign trade and public finance required the adoption of restrictive measures.
Equilibrium real effective exchange rate estimation for the Slovak economy

and the transition to floating exchange rate in 1998. The accumulated imbalances resulted, together with the increased uncertainty in foreign exchange market during the Asian and Russian crisis, in a sudden correction of the overvalued exchange rate that overshot the equilibrium level temporarily in 1999. The real exchange rate did not deviate for longer periods from the equilibrium levels during the period from 2000 to the moment of euro adoption with the exception of temporary increased uncertainty of the foreign exchange market agents related to the parliamentary elections in 2002 and to a greater extent also in 2006. The real exchange rate became overvalued temporarily after the fixation of Slovak crown’s nominal exchange rate to euro in 2009 that had taken into account also the future equilibrium appreciation founded on the continuing real convergence of the Slovak economy. This was caused by the deteriorated development of economic fundamentals accompanying the culmination of the financial crisis impacts on the real economy. However, the absence of an option to react more quickly to the mentioned situation with a nominal exchange rate depreciation did not pose a longer term jeopardy for the domestic producers price competitiveness because the real exchange rate returned below its equilibrium level in the next year already due to the negative inflation differential relative to the relevant trading partners of Slovakia (the initial depreciation of the currencies of several trading partners of Slovakia, that are situated outside the euro area, and their following appreciation relative to the euro played its role too). Continuing positive development of the equilibrium exchange rate macroeconomic determinants after the predicted recovery of domestic economy and foreign demand from the impacts of economic crisis enables certain real appreciation without any threat to the competitiveness and balance of the Slovak economy also in the future.

It is useful to compose an equilibrium band from the results of the alternative approaches due to the uncertainty implied by the variety of possible approaches to the equilibrium exchange rate estimation. Exchange rate shifts inside the boundaries of the equilibrium band should not introduce imbalances to the economy. Graph 10 contains the equilibrium band based on the outputs of the BEER and FEER models.

The actual and equilibrium real effective exchange rate development over the past and forecasted period is summarised in Graph 10: Equilibrium real effective exchange rate band.
sed in the form of average year-on-year dynamics and cumulative appreciation in table 1.

Table 1: Average annual and cumulative real effective exchange rate appreciation

<table>
<thead>
<tr>
<th>%</th>
<th>average annual appreciation</th>
<th>cumulative appreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>REER</td>
<td>1.86</td>
<td>-</td>
</tr>
<tr>
<td>EREER</td>
<td>1.77</td>
<td>2.17</td>
</tr>
<tr>
<td>BEER Model 1</td>
<td>1.78</td>
<td>2.42</td>
</tr>
<tr>
<td>BEER Model 2</td>
<td>1.75</td>
<td>3.22</td>
</tr>
<tr>
<td>FEER</td>
<td>1.77</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Source: own calculations

REER – real effective exchange rate based on PPI-manufacturing
EREER – equilibrium real effective exchange rate
BEER Model 1, BEER Model 2, FEER – equilibrium effective exchange rate estimates obtained from the individual BEER and FEER models

Conclusion

Equilibrium real effective exchange rate estimate for the Slovak economy presented in this paper helps to evaluate the total impact of the development of nominal exchange rates against the currencies of Slovakia’s relevant trading partners, domestic and foreign inflation on the price competitiveness of domestic companies over time. The appreciating trajectory of the equilibrium exchange rate is related to the gradual nominal convergence of the Slovak economy to the developed countries that is founded on the macroeconomic fundamentals development reflecting ongoing real convergence. On the basis of the equilibrium real exchange rate forecast resulting from the predicted values of the fundamentals, further real appreciation can be expected corresponding with the continuing convergence of the economy also in the future without jeopardising the economic balance. The real exchange rate misalignments contribute to the identification of expansive or restrictive monetary policy influence on the economy. Multiple alternative approaches frequently used in the literature, which differ in the definition of equilibrium, are applied in order to obtain objective equilibrium exchange rate estimates. The BEER approach defines the equilibrium real exchange rate as a function of relevant macroeconomic variables. According to the FEER approach,
the equilibrium exchange rate trajectory is such trajectory, which contributes to achieving both the internal (output at the level of its potential) and external (sustainable development of the current account of the balance of payments) balance of the economy.
References

Appendix

Estimate of the BEER model parameters:

Model 1

Date: 20/04/11 Time: 15:33
Sample (adjusted): 1995Q3 2010Q4
Included observations: 62 after adjustments
Trend assumption: Linear deterministic trend
Series: LOG(REER_SA) LOG(DPRODL) LOG(I_Y) NX_Y
Lags interval (in first differences): 1 to 9

Unrestricted Cointegration Rank Test (Trace)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.536392</td>
<td>81.86559</td>
<td>47.85613</td>
<td>0.0000</td>
</tr>
<tr>
<td>At most 1 *</td>
<td>0.292098</td>
<td>34.20524</td>
<td>29.79707</td>
<td>0.0146</td>
</tr>
<tr>
<td>At most 2</td>
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<td>15.49471</td>
<td>0.1228</td>
</tr>
<tr>
<td>At most 3</td>
<td>0.009743</td>
<td>0.607054</td>
<td>3.841466</td>
<td>0.4359</td>
</tr>
</tbody>
</table>

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Max-Eigen Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
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<td>27.58434</td>
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<tr>
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<tr>
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</tr>
<tr>
<td>At most 3</td>
<td>0.009743</td>
<td>0.607054</td>
<td>3.841466</td>
<td>0.4359</td>
</tr>
</tbody>
</table>

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values
Unrestricted Cointegrating Coefficients (normalized by $b^*S_11b=I$):

<table>
<thead>
<tr>
<th>LOG(REER_SA)</th>
<th>LOG(DPRODL)</th>
<th>LOG(I_Y)</th>
<th>NX_Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-95.64102</td>
<td>52.04264</td>
<td>12.49434</td>
<td>0.975004</td>
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<tr>
<td>-39.53000</td>
<td>39.96487</td>
<td>9.029293</td>
<td>-0.174254</td>
</tr>
<tr>
<td>-7.825965</td>
<td>6.006009</td>
<td>-24.05378</td>
<td>-0.215257</td>
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<td>-11.64646</td>
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<td>-11.75334</td>
<td>-0.227234</td>
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</table>

Unrestricted Adjustment Coefficients (alpha):

<table>
<thead>
<tr>
<th>D(LOG(REER_SA))</th>
<th>D(LOG(DPRODL))</th>
<th>D(LOG(I_Y))</th>
<th>D(NX_Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.006551</td>
<td>-0.000211</td>
<td>0.002941</td>
<td>0.000147</td>
</tr>
<tr>
<td>0.000622</td>
<td>-0.001564</td>
<td>-0.000478</td>
<td>-0.001033</td>
</tr>
<tr>
<td>-0.006044</td>
<td>-0.012492</td>
<td>0.004565</td>
<td>0.000405</td>
</tr>
<tr>
<td>-0.562993</td>
<td>0.572943</td>
<td>0.247936</td>
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</tr>
</tbody>
</table>

1 Cointegrating Equation(s): Log likelihood 427.8096

Normalized cointegrating coefficients (standard error in parentheses)

<table>
<thead>
<tr>
<th>LOG(REER_SA)</th>
<th>LOG(DPRODL)</th>
<th>LOG(I_Y)</th>
<th>NX_Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000000</td>
<td>-0.544146</td>
<td>-0.130638</td>
<td>-0.010194</td>
</tr>
<tr>
<td>(0.03855)</td>
<td>(0.05669)</td>
<td>(0.00135)</td>
<td></td>
</tr>
</tbody>
</table>

Adjustment coefficients (standard error in parentheses)

<table>
<thead>
<tr>
<th>D(LOG(REER_SA))</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.562570</td>
<td>(0.18309)</td>
</tr>
</tbody>
</table>
Model 2

Date: 20/04/11 Time: 15:34
Sample (adjusted): 1996Q1 2010Q4
Included observations: 60 after adjustments
Trend assumption: Linear deterministic trend
Series: LOG(REER_SA) LOG(DPRODL) NX_Y LOG(AFD_X)
Lags interval (in first differences): 2 to 2, 5 to 7, 10 to 11

Unrestricted Cointegration Rank Test (Trace)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.413861</td>
<td>57.24856</td>
<td>47.85613</td>
<td>0.0051</td>
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<tr>
<td>At most 1</td>
<td>0.281725</td>
<td>25.19662</td>
<td>29.79707</td>
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<tr>
<td>At most 2</td>
<td>0.069731</td>
<td>5.342447</td>
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<tr>
<td>At most 3</td>
<td>0.016620</td>
<td>1.005588</td>
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<td>0.3160</td>
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</table>

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Max-Eigen Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
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<tr>
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<tr>
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<td>At most 3</td>
<td>0.016620</td>
<td>1.005588</td>
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</table>

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegrating Coefficients (normalized by b**S11*b=I):

<table>
<thead>
<tr>
<th>LOG(REER_SA)</th>
<th>LOG(DPRODL)</th>
<th>NX_Y</th>
<th>LOG(AFD_X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.58509</td>
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<td>-0.199606</td>
<td>3.835532</td>
</tr>
<tr>
<td>1.142071</td>
<td>19.24319</td>
<td>-0.452284</td>
<td>2.166891</td>
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<tr>
<td>-15.52295</td>
<td>9.210660</td>
<td>0.138975</td>
<td>8.163744</td>
</tr>
<tr>
<td>-28.24801</td>
<td>11.87183</td>
<td>0.047635</td>
<td>1.606591</td>
</tr>
</tbody>
</table>
Unrestricted Adjustment Coefficients (alpha):

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient 1</th>
<th>Coefficient 2</th>
<th>Coefficient 3</th>
<th>Coefficient 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(LOG(REER_SA))</td>
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<td>0.001980</td>
<td>-0.001461</td>
<td>-0.000262</td>
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<tr>
<td>D(LOG(DPRODL))</td>
<td>0.002860</td>
<td>0.000413</td>
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<tr>
<td>D(NX_Y)</td>
<td>0.196714</td>
<td>1.001898</td>
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</tr>
<tr>
<td>D(LOG(AFD_X))</td>
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<td>0.002335</td>
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</tr>
</tbody>
</table>

1 Cointegrating Equation(s): Log likelihood 298.3769

Normalized cointegrating coefficients (standard error in parentheses)

| Term      | Coefficient 1 | Standard Error
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LOG(REER_SA)</td>
<td>1.000000</td>
<td>(0.18234)</td>
</tr>
<tr>
<td>LOG(DPRODL)</td>
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<td>(0.00404)</td>
</tr>
<tr>
<td>NX_Y</td>
<td>-0.008463</td>
<td>(0.07799)</td>
</tr>
<tr>
<td>LOG(AFD_X)</td>
<td>0.162625</td>
<td></td>
</tr>
</tbody>
</table>

Adjustment coefficients (standard error in parentheses)

| Term      | Coefficient 1 | Standard Error
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>D(LOG(REER_SA))</td>
<td>-0.222137</td>
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Estimate of the FEER model parameters:

Export equation

Date: 20/05/11 Time: 13:59
Sample (adjusted): 1995Q4 2010Q4
Included observations: 61 after adjustments
Trend assumption: Linear deterministic trend
Series: LOG(X00) LOG(M_EF) LOG(REER_SA) LOG(DPRODL)
Lags interval (in first differences): 1 to 1, 3 to 3, 6 to 10

Unrestricted Cointegration Rank Test (Trace)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.497704</td>
<td>78.32134</td>
<td>47.85613</td>
<td>0.0000</td>
</tr>
<tr>
<td>At most 1 *</td>
<td>0.352394</td>
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<td>29.79707</td>
<td>0.0077</td>
</tr>
<tr>
<td>At most 2</td>
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<td>15.49471</td>
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</tr>
<tr>
<td>At most 3</td>
<td>2.18E-05</td>
<td>0.001331</td>
<td>3.841466</td>
<td>0.9698</td>
</tr>
</tbody>
</table>

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values
### Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Max-Eigen Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.497704</td>
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<td>0.0004</td>
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</tr>
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<td>At most 3</td>
<td>2.18E-05</td>
<td>0.001331</td>
<td>3.841466</td>
<td>0.9698</td>
</tr>
</tbody>
</table>

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

### Unrestricted Cointegrating Coefficients (normalized by b*S11*b=I):

<table>
<thead>
<tr>
<th>LOG(X00)</th>
<th>LOG(M_EF)</th>
<th>LOG(REER_SA)</th>
<th>LOG(DPRODL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-18.41160</td>
<td>33.15593</td>
<td>28.50678</td>
<td>-30.75769</td>
</tr>
<tr>
<td>-10.27195</td>
<td>-17.43445</td>
<td>-0.543044</td>
<td>77.00415</td>
</tr>
<tr>
<td>-12.43201</td>
<td>18.93416</td>
<td>-2.957278</td>
<td>-10.31825</td>
</tr>
</tbody>
</table>

### Unrestricted Adjustment Coefficients (alpha):

<table>
<thead>
<tr>
<th>D(LOG(X00))</th>
<th>D(LOG(M_EF))</th>
<th>D(LOG(REER_SA))</th>
<th>D(LOG(DPRODL))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004296</td>
<td>-0.002835</td>
<td>-0.001409</td>
<td>-0.008217</td>
</tr>
<tr>
<td>0.006454</td>
<td>-0.004181</td>
<td>-0.005394</td>
<td>0.003700</td>
</tr>
<tr>
<td>0.000941</td>
<td>0.002879</td>
<td>-0.004758</td>
<td>-0.000740</td>
</tr>
<tr>
<td>0.000134</td>
<td>4.97E-05</td>
<td>-2.20E-05</td>
<td>1.66E-05</td>
</tr>
</tbody>
</table>

### 1 Cointegrating Equation(s):

Log likelihood 673.3689

Normalized cointegrating coefficients (standard error in parentheses)

<table>
<thead>
<tr>
<th>LOG(X00)</th>
<th>LOG(M_EF)</th>
<th>LOG(REER_SA)</th>
<th>LOG(DPRODL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000000</td>
<td>-0.994358</td>
<td>1.311145</td>
<td>-2.404843</td>
</tr>
<tr>
<td></td>
<td>(0.26784)</td>
<td>(0.33543)</td>
<td>(0.76205)</td>
</tr>
</tbody>
</table>

Adjustment coefficients (standard error in parentheses)

<table>
<thead>
<tr>
<th>D(LOG(X00))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.092568</td>
</tr>
<tr>
<td>(0.12018)</td>
</tr>
</tbody>
</table>
Import equation

Date: 20/05/11 Time: 14:01
Sample (adjusted): 1996Q4 2010Q4
Included observations: 57 after adjustments
Trend assumption: Linear deterministic trend
Series: LOG(M00) LOG(DD00) LOG(REER_SA) LOG(X00)
Lags interval (in first differences): 2 to 2, 9 to 14

Unrestricted Cointegration Rank Test (Trace)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.568044</td>
<td>68.42890</td>
<td>47.85613</td>
<td>0.0002</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.277916</td>
<td>20.58130</td>
<td>29.79707</td>
<td>0.3843</td>
</tr>
<tr>
<td>At most 2</td>
<td>0.026430</td>
<td>2.021281</td>
<td>15.49471</td>
<td>0.9947</td>
</tr>
<tr>
<td>At most 3</td>
<td>0.008638</td>
<td>0.494496</td>
<td>3.841466</td>
<td>0.4819</td>
</tr>
</tbody>
</table>

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Max-Eigen Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.568044</td>
<td>47.84760</td>
<td>27.58434</td>
<td>0.0000</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.277916</td>
<td>18.56002</td>
<td>21.13162</td>
<td>0.1103</td>
</tr>
<tr>
<td>At most 2</td>
<td>0.026430</td>
<td>1.526784</td>
<td>14.26460</td>
<td>0.9978</td>
</tr>
<tr>
<td>At most 3</td>
<td>0.008638</td>
<td>0.494496</td>
<td>3.841466</td>
<td>0.4819</td>
</tr>
</tbody>
</table>

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegrating Coefficients (normalized by $b^*S_{11}b=I$):

<table>
<thead>
<tr>
<th>LOG(M00)</th>
<th>LOG(DD00)</th>
<th>LOG(REER_SA)</th>
<th>LOG(X00)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-27.45307</td>
<td>13.25784</td>
<td>21.29903</td>
<td>13.52574</td>
</tr>
<tr>
<td>61.6304</td>
<td>-40.93587</td>
<td>34.55942</td>
<td>-45.62911</td>
</tr>
<tr>
<td>31.43491</td>
<td>-53.52580</td>
<td>23.56635</td>
<td>-10.27019</td>
</tr>
<tr>
<td>-26.66324</td>
<td>27.37021</td>
<td>-13.52547</td>
<td>1794451</td>
</tr>
</tbody>
</table>
**Unrestricted Adjustment Coefficients (alpha):**

<table>
<thead>
<tr>
<th>D(LOG(M00))</th>
<th>0.009206</th>
<th>-0.008553</th>
<th>0.003279</th>
<th>-0.002454</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(LOG(DD00))</td>
<td>0.001478</td>
<td>-0.001567</td>
<td>0.003118</td>
<td>-0.000523</td>
</tr>
<tr>
<td>D(LOG(REER_SA))</td>
<td>-0.010720</td>
<td>-0.003970</td>
<td>-0.000196</td>
<td>0.000496</td>
</tr>
<tr>
<td>D(LOG(X00))</td>
<td>0.003962</td>
<td>-0.001764</td>
<td>0.000520</td>
<td>-0.003062</td>
</tr>
</tbody>
</table>

**1 Cointegrating Equation(s):**

<table>
<thead>
<tr>
<th>Normalized cointegrating coefficients (standard error in parentheses)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOG(M00)</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1.000000</td>
</tr>
<tr>
<td>(0.19629)</td>
</tr>
</tbody>
</table>

**Adjustment coefficients (standard error in parentheses):**

<table>
<thead>
<tr>
<th>D(LOG(M00))</th>
<th>-0.252728</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.19990)</td>
<td></td>
</tr>
</tbody>
</table>

**Endogenous variables fit in the FEER model without residuals**
Endogenous variables fit in the FEER model after the inclusion of smoothed residuals

Residuals from the individual foreign trade equations and their paths smoothed with the HP-filter

4 Extrapolation of the actual real exchange rate development based on the equilibrium appreciation estimate obtained from the average results of the two BEER models is used to compute the residuals in the forecasted period.